Topic

3 Using Mathematical Models to Investigate Planetary Habitability

Considering a Real World Problem

Deforestation and Urban Heat Islands

The Problem

Figure 3.1. View of New York City. Produced by World Surveillance Inc. using satellite imagery, aerial photography, and computer enhancement. The image shows heat levels from very hot red to cool blue. (satellite.rsat.com/rsat/apps/uhi_app/uhi_app/html)

Earth's surface features are continually changing. In our planet's ancient past, natural events caused the global mean temperature to cool to such an extreme that ice and snow covered much of the earth's surface, producing several Ice Ages.

But today, there is another variable to consider as a forcing on the planetary landscape and habitability: human civilization.

Land is cleared for agricultural production and to raise cattle. Loggers cut down trees for commercial profit from products manufactured with wood. This process of deforestation is decreasing the world's forests.

Tropical forests are particularly impacted. Some reports estimate that these areas will disappear in 100 years if the current rate of deforestation continues (NASA, 1998). The Brazilian Congress is voting to clear 50% the Amazon forest for agriculture and cattle. They believe this plan will improve the country's poor economy and enhance personal economic prosperity of its citizens. What effects might deforestation have on Earth's global and local temperatures?

Another way humans are changing Earth's surface is by building structures on the landscape. In the 20th century, great cities have developed. Areas once covered with plants and trees are now sites of skyscrapers, buildings and roads. These large cities are associated with a phenomenon called *Urban Heat Islands*, where temperatures warm due to built-up environments. Why would temperatures increase as a city grows?

Activity A

Finding a Mathematical Description of a Physical Relationship

Overview

Your investigations in the laboratory have shown you the general relationship that the farther away an object is from an energy source, the lower its temperature and the less the energy that reaches it. This qualitative description is useful if we want to know which of two objects will be hotter or colder depending upon their relative positions from the energy source. But what if we want a more specific answer? *How* hot or cold will an object be if placed at a specific distance from an energy source? Exactly *how much* energy reaches the object? To get answers we need a more *quantitative* description of the relationship between these factors. Only when we have an exact description of the relationship are we able to make quantitative predictions.

The purpose of this activity is to obtain data necessary to precisely describe the relationships between energy arriving at an object located at a certain distance from an energy source, and its resulting temperature. Solar cells will be used to produce voltage that is directly proportional to the intensity of a light source. A volt meter will be used to record this voltage output, effectively measuring the relative brightness of the light. When determining the temperature of the object, be sure to use identical models. Teams will perform two investigations:

Experiment 1. Examine the intensity of light at different distances from the energy source.

Experiment 2. Examine the temperatures of an object at these various distances.

Before beginning these experiments, determine team member roles and prepare an experimental proposal. The data collected will be analyzed to produce a mathematical description (an equation) that relates the factors. In a later activity, these mathematical descriptions will be used to create a mathematical model.

Learning Objectives

- ✓ Make measurements with a volt meter.
- ✓ Create a graph using a spreadsheet.
- ✓ Determine the best fit line for data using a spreadsheet.
- ✓ Quantitatively relate distance from a source to energy incident on an object.
- ✓ Quantitatively relate temperature of an object to the distance from a energy source.
- ✓ Use the derived relationships to predict observed values.

Relevance

Often the process or phenomena that the scientist tries to model in a laboratory is too complicated, expensive or hazardous to simulate with a physical model. Increasingly, scientists turn to mathematical models to perform these simulations. Such models are based on mathematical descriptions of known processes constrained by physical limits observed in nature. These models are increasingly important to researchers, hence you should become familiar with how they are developed and the means by which they can be useful tools.

Finding a Mathematical Description of a Physical Relationship

Materials

Light sources (150 watt light bulbs)
Support for light source
Solar cell/sunphotometers
Digital thermometers
1 volt meter
3 small plastic cups (to hold identical models)
Materials to make identical surfaces for each model
Computers loaded with Microsoft Excel software for the second part of this activity

Methods

Preliminary Discussion and Planning

1. Discuss: Modeling experiments designed to produce the greatest temperature difference between a hot and a cold planet.
2. Challenge: Design a physical model using the materials listed above to quantitatively determine the effect of distance from an energy source on the intensity of energy reaching that distance, and upon the temperature at that point.
3. Competition: Determine the mathematical relationships that best describe the effect you discover in step 2 above.
4. Organize into investigative teams of about five students and determine the role of each team member. Complete and submit the Investigation sheet: Team Members.

Experimental Design

1. Prepare your team's research proposal by completing and submitting the Experimental Design Proposal.
2. Prepare the experimental plan by completing and submitting the Methodology for a Controlled Experiment.

Experimentation and Observation

1. Set up the two experiments based on your proposal.
2. Each team will follow a common protocol by taking light intensity and temperature measurements at the same distances for at least seven different distances.
3. Data is recorded on the Data Sheet: Physical Model Experiments.
4. Complete the Data Sheet: Experimental Results.
5. Use a spreadsheet program on a computer to graph your results.

Data Analysis, Comparisons and Consensus

1. Review the data collected by your team for both experiments.
2. Each sub-team should plot its own data and from that plot try to determine the mathematical relationship between the two variables.
3. Each sub-team should then use the spreadsheet program, Microsoft Excel, to re-plot its data, and then use the features of the spreadsheet to re-evaluate the mathematical relationship between their variables.
4. Complete the Data Sheet: Experimental Results.
5. Coordinate your team's presentation to the class.
6. Contribute to the class discussion of results.
7. Answer the questions at the conclusion of experiments and data analysis (pg. 76). This is an individual assessment of what you learned in the modeling experiments. All responses should be shared among the team members to gain a collective understanding of your results.

Investigation Notebook

1. Investigation: Team Members
2. Investigation: Experimental Design Proposal
3. Investigation: Methodology for a Controlled Experiment
4. Data Sheet: Physical Model Experiments
5. Data Sheet: Experimental Results
6. Questions: Finding a Mathematical Description of a Physical Relationship

Investigation Activity A

Team Members

Before planning and carrying out your experiment, decide who will perform each research role. Review the responsibilities and select team member roles.

Lead Researcher ______________________________

Responsibilities:

- Organize activities of team.
- Ensure all members are contributing productively.
- Communicate with other groups to verify experimental procedures and results, and criticize the results of your experiments.
- Make suggestions to improve your experimental procedure.
- Initiate the repetition of an experiment if necessary.
- Keep notes on the following:
    1. How could your experimental procedure be improved?
    2. What materials do you lack in order to make this a more effective experiment?

Materials and Data Managers

Intensity of Light	Temperature of Model
______________________________	______________________________

Responsibilities:

- Acquire materials for setting up experiment.
- Clear a sufficiently large workspace at a lab table.
- Keep organized records of all experimental measurements.
- Organize and return equipment at the end of class.
- Organize and submit the group's written material.

Experimental Communications

Intensity of Light	Temperature of Model
______________________________	______________________________

Responsibilities:

- Carefully note all steps of the experimental procedures.
- Create diagrams of the experimental set-up.
- Create graphs of the experimental results.
- Coordinate the team presentation of the results and analysis.

Investigation Activity A

Experimental Design Proposal

To prepare for the experiment, start by proposing a hypothesis. Try to anticipate the limitations of your experiment. Review your hypothesis and limitations with your instructor.

Hypothesis

How do you expect changing distance to affect the intensity of the light reaching a point at that distance?

How do you expect changing the distance will affect the surface temperature of your model?

Experimental Limitations

How do the laboratory materials available for this experiment differ from the objects they are simulating?

Experimental Expectations

Do you expect changing distance to have the same effect upon temperature and intensity of the light reaching a point?

Methodology for a Controlled Experiment

Design two experiments to be performed by the sub-teams previously selected. The first team should investigate the relationship between distance from a light source and the intensity of light at that distance. The second experiment should investigate the relationship between the distance of an object from the light source and the temperature of the object at that point.

Laboratory Materials	**Simulated Object**
Light source	Sun
Plastic cup	Planet
Gravel, water, clay, sand, other textured materials	Surface of the planet, e.g., continents, oceans
Additional Materials	
Meter sticks, thermometers, volt meters, and solar cells	

Discuss the following questions before carrying out your experiment. Wait until your experiment is in progress to fill in your answers.

Experimental Procedure

Indicate the steps you plan to take in order to conduct your experiment.

Analysis of Experimental Variables

Variables
List all the quantities that could change during the experiment.

Independent Variable
Which variable can the experimenter purposely change?

Dependent Variable
Which variable will respond to change in the independent variable?

Experimental Controls
Which factors will not change during the experiment?

Physical Model Experiments

Record your data at a minimum of seven different distances in table 3.1 below.

Distance (cm)	Intensity Measurements (volts)			Temperature Measurements	
	Source On	Source Off	Distance Intensity	ºC	K

Table 3.1. Data from Physical Model Experiments.

To simplify your calculations and models in this activity, you can "normalize" obtained values using table 3.2 below. Normalizing requires that you select one of your values, make it your standard unit by assigning it a value of 1. This can be done by dividing that value by itself. To change all measurements to the same unit or scale, you simply divide each of the other values of the same variable by this standard value.

For example, if you had distances of 1 m, 4 m, 5 m, 10 m, 12 m, and 20 m, select any one of these values as your standard unit. For example, if you select 5 m, to normalize all these distances you would divide this and all other values by 5 m. This would give normalized distances of 0.2, 0.8, 1, 1.4, and 4 respectively. The first distance is 0.2 of the standard unit. In other words, it is 0.2 of the standard distance.

Take your third set of values from the table as your standard values. Divide each distance by this selected standard value. Record results in table 3.2 below. Also normalize observed intensity and the temperature in degrees Kelvin and record them in the table.

Normalized Distance (Distance Units)	**Normalized Intensity** (Intensity Units)	**Normalized Temperature** (K)

Table 3.2. Normalized Values from Observations.

Experimental Results

Based on the data collected for the effect of distance on the intensity of light and on the temperature, report your experimental results. Plot your data and draw a best-fit line or curve, whichever is appropriate, for the values. From the best-fit line or curve, develop a mathematical equation to describe that line or curve. Compare the results of your two experiments and explain how these preliminary results relate to your original hypothesis.

Independent Variable: ______________________________

Dependent Variable: Intensity of light (volt meter reading) at a specific point.

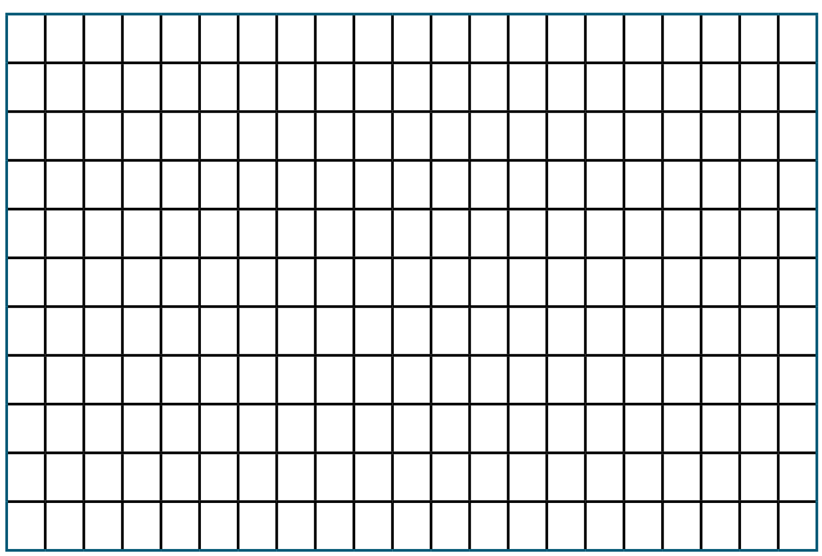
Experiment 1 – Physical Model: Effect of Distance Upon Intensity of Light

Experimental Conditions

Describe your experimental setup.

Experimental Results

Label the axes below, plot data that you recorded in table 3.1 and describe the relationship you observe as best as you can.



Experiment 2 – Physical Model: Effect of Distance Upon Temperature

Experimental Conditions

Describe your experimental setup.

Experimental Results

Plot your data from table 3.1 and describe the relationship you observe as best as you can.

Preliminary Results from Experiments 1 and 2

Compare the relationships between distance and the two conditions. Explain how these preliminary results bear upon your original hypothesis.

Can you quantitatively describe either of the relationships based upon the graphs you have drawn? Why or why not?

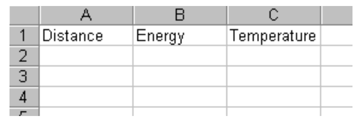
Deriving Quantitative Relationships with a Spreadsheet

When a relationship between variables is not linear, it can be difficult to determine the exact mathematical relationship by simply examining data. A computer spreadsheet program can often help produce more quantitative results. It allows the user to easily manipulate, graph and analyze relationships using large amounts of numerical data. Use Microsoft Excel software to plot your data from the physical model experiment. Does this tool improve your ability to mathematically describe relationships among variables?

Producing a Graph Using a Spreadsheet

- On a computer, start Excel. Have the data from your experiment ready.

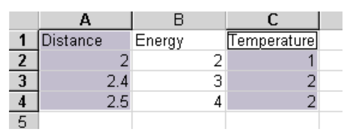
- Using the normalized values from your table 3.2, enter information in columns beginning with cell A1. There should be 3 columns in your spreadsheet – distance, energy and temperature. Label the first cell in each column with a title for that column. If you are entering temperatures in Celsius, be sure to convert them to the Kelvin scale in the spreadsheet.



- When the table is complete, begin analyzing the data. Sometimes, one can see relationships by examining numerical values. But it is usually easier to see them by plotting data points on a graph and studying the shape of the graph.

- Using the left button on the mouse, click on your title in the first cell of the first data column and hold down the mouse button. Drag the cursor to the last cell with data in this column and release the mouse button. All the data in the column should appear selected.

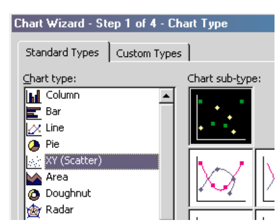
- Carefully, press and hold down the CTRL key on the keyboard. Then click on the title of the second data column you wish to plot and hold down the mouse button. With the CTRL key and the mouse button still pressed, drag the cursor down to the last cell with data in this column. Both columns should now appear selected.



Note: The data in the illustration above are examples. Use the data from your experiment to construct your own spreadsheet

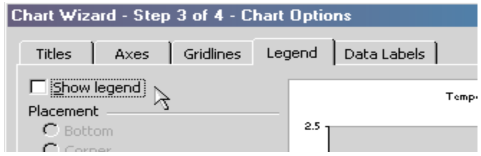
- On the toolbar located at the top of the spreadsheet, click on the Chart Wizard button – it is a multicolored icon that looks like a bar graph. Alternatively, you can click on the **Insert** menu, and then on **Chart**.

- A dialog box should appear to guide you through the graphing process. At the first step of the process, select **XY Scatter** as the chart type, choose the first sub type available (no line is drawn through the data.), and click on the **Next** button.



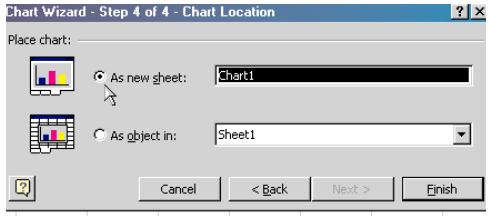
- At the second step, you will see a preview of your graph. Click **Next** to go to step 3.

- In step 3 - Chart Options, go to the **Legend** tab and remove the check mark on **Show legend**.



- Go to the **Titles** tab and fill in values for your graph title and the x and y axes. Be sure to include the units in axis titles. Click **Next**.

- In step 4 - Chart Location, select **As new sheet** and provide a title. For example: Energy vs. Distance. Click **Finish**. The completed graph appears on a new sheet by itself.



Examine the graph that you have produced.

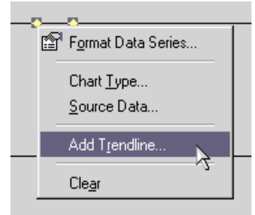
1. Describe the relationship between the two variables you have graphed.

2. What do you think is the most significant characteristic of the graph?

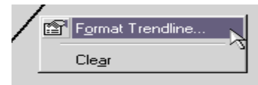
3. Does the graph agree with the graph you drew by hand?

4. Does the graph ever touch the horizontal (distance) axis?

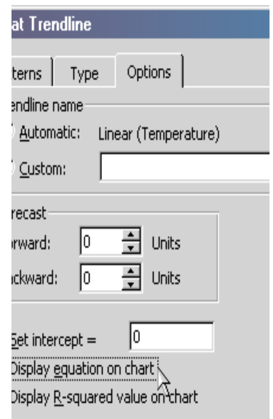
Produce a trendline (the line that best fits your data) for your graph.



- Using the right button on the mouse, click on any data point on the graph. On the menu that appears, select **Add Trendline…**
- Select the trendline shape that best resembles your plotted data. Ignore upwards and downwards extremes. Base your selection on the general shape. If the shape does not fit your data well, try others until you get what appears to be the best fit.



- After selecting your trendline, right click on the line. Select **Format Trendline...**, and then click on the **Options** tab and select **Display equation on chart**. Click on **OK**.
- The graph itself may obscure the equation of your trendline. You can click on and drag the equation to a clear area on the graph.



Examine the trendline you have produced.

1. How well does your trendline "fit" your data? Why do you think this is so?

2. What is the equation of your trendline?

3. How does this equation compare with the relationship you described earlier in Activity A of Topic 3?

4. How does this equation compare with your hand drawn graphical result?

5. Give a physical explanation of why this relationship exists in this form.

Questions Activity A

Finding a Mathematical Description of a Physical Relationship

Answer the following questions on your own in complete sentences. You will need to use additional sheets of paper to answer all the questions completely.

1. How does the experiment you performed here differ from the hot and cold planets experiment performed in Topic 2?

2. Did the changing distance affect the temperature and the intensity of the light as you predicted? Explain.

3. What advantages did the spreadsheet program offer over analyzing your data by hand?

4. Do your final equations accurately describe the relationships between distance and temperature, and distance and intensity? Why or why not?

5. Are these two relationships sufficient to model planets in the solar system? Explain.

6. Two scientists perform different physical model experiments to investigate a particular phenomenon. They derive different equations to explain the phenomenon. How can they decide which equation is a better model?

7. An architect wants to know if the design for a 10-story building for downtown Los Angeles will be able to withstand a strong earthquake. What options are there for testing the design? What are the advantages and disadvantages of the options?

Activity B

Making a Simple Mathematical Model

Overview

You will use your observed results for the relationships between distance and energy reaching an object and distance and temperature of an object to make your own simple mathematical model. Your model will allow you to change the luminosity of the light source and the distance to the light source. You can calculate the energy reaching the object and its black body temperature. Produce simulations of Earth, Venus and Mars to obtain their theoretical black body temperatures. Compare these results to the actual planetary temperature values. Upon completing the experiments, discuss the significance of this comparison and the role of models in science as well as possible improvements that could be made to your model.

Your investigations have shown a general relationship between the distance from the sun to a planet and the resulting planetary surface temperature – the farther away the planet, the lower its temperature. In Activity A, you derived a quantitative relationship between distance and intensity of energy. In Topic 2, you used the Radiation Balance Model to simulate the hot and cold planet experiment. How well did the model recreate these values and relationships? How can a mathematical computer model be created to model the features of a system?

A simple computer model will be created to determine the black body surface temperature of planets in the solar system. You will build this model on a computer using Microsoft Excel software. Mathematical equations will be used to modify given inputs to obtain desired results. The model inputs will be luminosity of the sun and the distance between the sun and a planet.

Learning Objectives

- ✓ Make a spreadsheet model using Excel to determine the temperature of an object given its distance from an energy source.
- ✓ Compare theoretical and observed black body temperatures for a planet.
- ✓ State three advantages of using mathematical models.
- ✓ Describe three disadvantages of using mathematical models.

Relevance

Even the most complicated mathematical models used by scientists today began with a few simple equations representing real world phenomena. You have seen how it is possible to derive a mathematical equation to describe relationships in experimental data. The next step is to use that relationship in a model to make predictions about real world situations. As more relationships are included in a model, along with inter-relationships and feedbacks, it becomes exceedingly complicated, but the basic equations remain at its core. Like scientists involved in climate modeling studies, you will use the relationships that you derived for your model of the solar system and evaluate the model's effectiveness.

Investigation Activity B

Making a Simple Mathematical Model

Materials

A computer loaded with Microsoft Excel software, and the equations derived by you in this topic's Activity A.

Methods

Preliminary Discussion and Preparation

1. Discuss your expectations for your mathematical model with the class.
2. Familiarize yourself with the basic features of an Excel spreadsheet by reading and answering the questions in the Investigation section: Learning Excel – Applying Features of a Spreadsheet Program to Create a Simple Mathematical Model.

Experimentation and Modeling

1. Follow the instructions to make your preliminary model and perform the experiments in the Investigation sheet: Creating a Simple Computer Model of a Planetary System.
2. Enter your model data on the Data Sheet: Making a Simple Mathematical Model.

Data Analysis and Consensus

1. Compare the output of your model with the true values for Earth, Venus and Mars.
2. Evaluate the accuracy of your model.
3. Suggest ways of improving your model.
4. Discuss the advantages and disadvantages of using mathematical models.
5. To assess what you learned in the modeling experiments, complete the Investigation Questions.

Investigation Notebook

1. Investigation: Learning Excel
2. Investigation: Creating a Simple Computer Model of a Planetary System
3. Data Sheet: Making a Simple Mathematical Model
4. Data Sheet: Experimental Results
5. Questions: Making a Simple Mathematical Model

Investigation Activity B

Learning Excel

Applying the Features of a Spreadsheet Program to Create a Simple Mathematical Model

Start Excel on your computer. Once you begin the program, you will see a few blank worksheets that make up an Excel file, called a workbook, on the computer. Each sheet is divided into a series of lettered columns A, B, C, etc., and a series of numbered rows 1, 2, 3, etc. Each box, called a cell, in the spreadsheet is designated by its column and row identifier, such as A1, A5, B14 and so on. A cell can contain a value such as a word or a number, or it can contain a mathematical equation. If an equation is placed into a cell, the *result* of the equation is displayed in the cell and not the equation itself.

For example, a spreadsheet can be designed to determine the hypotenuse of a right triangle. The inputs would be the lengths of the two sides of the right triangle and the desired result would be the hypotenuse.

There are several ways to approach this problem. You could make a spreadsheet into which one set of values for the sides is entered and a single resulting hypotenuse is displayed. Or, a table of values can be entered for the lengths and a table of resulting hypotenuses will be created. The choice is yours.

In your Excel spreadsheet, enter the title for the first column: **Side A** in cell A1. In cell B1, enter the title: **Side B**. In cell C1, enter the title: **Hypotenuse**. In A2 and B2 you will eventually enter the values of the sides of your right triangle. In C2 you need to enter the equation that will determine the hypotenuse.

The Pythagorean theorem states that for a right triangle with sides a and b and hypotenuse c, $a^2 + b^2 = c^2$. Solving this for c yields:

Equation One: $$c = \sqrt{(a^2 + b^2)}$$

This equation must be entered into cell C2 in such a way that the spreadsheet can understand it. To raise a value to a power in a spreadsheet, the ^ symbol can be used (which is found on the number 6 key on the keyboard). a^2 would be written as a^2, b^2 as b^2, etc.

Remember that $\sqrt{a} = (a)^{1/2}$, therefore we can type the above equation into cell C2 as:

Equation Two: =(A2^2+B2^2)^0.5

Notice that for the sides we do not enter a^2, but rather A2^2 – referring to the location of the value and not the value itself. This way any value can be entered into cells A2 and B2 and the resulting hypotenuse obtained in cell C2. After setting up the equation in your spreadsheet be sure to save the file on your computer.

Try entering some sample values for two sides of your right triangle. After constructing a model, it is usually a good idea to test it with known values to see if they give the expected results. A common right triangle is the 3, 4 and 5 triangle. Insert 3 and 4 as your sides and check the resulting hypotenuse. Your results should look similar to the diagram below. Test your spreadsheet with some other known right triangles.

	A	B	C	D
1	Side A	Side B	Hypotenuse	
2	3	4	5	
3				
4				
5				
6				

What are the hypotenuses of the right triangles with the following sides?

Triangle One: Sides: 25, 15
Hypotenuse: ______

Triangle Two: Sides: 90, 120
Hypotenuse: ______

Investigation Activity B

Creating a Simple Computer Model of a Planetary System

Modeling the Planets of the Solar System

Your task in this activity is to produce a mathematical model in a spreadsheet that can determine the black body surface temperatures of the planets in our solar system. Use the luminosity (brightness) of the sun (L), and the average distance (D) between the planet and the sun as your inputs.

Your spreadsheet model should be able to determine the average intensity of the energy reaching each square meter of the planet's surface (a value known as the *solar constant*, S) and the resulting black body temperature of the planet's surface. (A *black body* is a theoretical object that will absorb all the energy that it receives and then emit all that energy back to its surroundings.)

The two equations you derived in Activity A will come into play here. Use the relationship for the distance between the energy source and a point, and the amount of energy reaching that point, to determine the solar constant of a planet at a certain distance from the sun. Use the relationship between this distance or the amount of energy reaching that point and the surface temperature to determine the surface temperature of the same planet.

Section 1. The Amount of Energy Reaching a Planet

The light from the sun radiates outwards in all directions, illuminating a sphere centered about the sun. The total luminosity of the sun, L_{sun}, is currently known to be 3.845×10^{26} watts. Normalize this value and call it 1.00. The intensity of energy reaching the earth at the top of the atmosphere (the solar constant) has been measured to be 342 watts/meter^2. The earth is one astronomical unit from the sun. An astronomical unit (AU) is a convenient unit of distance when working with objects in orbit around the sun. 1 AU is the average distance between Earth and the sun.

Use these values and your distance relationship to write an equation for the spreadsheet that will enable you to determine the solar constant at any distance from a sun with a given luminosity, particularly at the positions of the nine planets in the solar system.

Section 2. The Black Body Surface Temperature of the Planet

If the earth were a black body located in its current orbit around our sun, its surface temperature would be about 6ºC (279 K). Use this value and the relationship from Activity A that describes the effect of distance or the amount of incoming energy upon the temperature of an object. Write an equation for your spreadsheet that will determine the black body temperature of a planet at a specific distance from the sun.

Section 3. Spreadsheet Model of the Solar System

Using the Reference: Planetary Distances from the Sun (on page 83) and the equations described in Sections 1 and 2 above, construct a spreadsheet model to determine the black body surface temperature of any planet in the solar system.

Your spreadsheet should allow you to input the following values:

1. the luminosity of the sun (L)
2. the distance of the planet from the sun (D).

The spreadsheet should then output the following values:

1. the solar constant (S) for that planet
2. the black body surface temperature

Planetary Distances from the Sun

Planet	Average Distance from the Sun (Astronomical Units)
Mercury	0.3870
Venus	0.7233
Earth	1
Mars	1.5234
Jupiter	5.2033
Saturn	9.5376
Uranus	19.1914
Neptune	30.0693
Pluto	39.4818

Table 3.3. Planetary distances from the sun expressed in astronomical units.

Data Sheet Activity B

Making a Simple Mathematical Model

Task 1

Using your spreadsheet model from Section 3 (above), determine the solar constant for each planet and the corresponding black body temperature of the planetary surface. Record results in table 3.4 below.

Planet	D = Average Distance from the sun (AU)	S = Solar Constant (watts/meter^2)	Black Body Temperature		
			(K)	(C)	(F)
Mercury	0.3870				
Venus	0.7233				
Earth	1				
Mars	1.5234				

Table 3.4. Predictions from Spreadsheet Model.

Task 2

Your instructor will provide you with the accepted values for these quantities. Fill in these values in the appropriate spaces in table 3.5 below.

Planet	S = Solar Constant (watts/meter^2)		% Error	Surface Temperature (°C)		% Error
	Model Value	Accepted Value		Model Value	Accepted Value	
Mercury						
Venus						
Earth						
Mars						

Table 3.5. Comparing Model Results with True Values.

Data Sheet Activity B

Experimental Results

1. How do the model and accepted values for the solar constant compare? Do you notice a general trend with these comparisons?

2. How do the model and accepted values for the surface temperature of the planets compare? Do you notice a general trend with these comparisons?

3. Did your model do a better job simulating the solar constant or the surface temperature? Why do you think this is so?

4. Which characteristic of your model would you most like to see improved? How would you go about making this improvement?

Making a Simple Mathematical Model

Answer all of the following questions on your own and in complete sentences. Use additional sheets of paper if necessary.

1. What problems did you encounter while trying to construct your spreadsheet model of the solar system?

2. How did you overcome these problems?

3. Does your model produce results that agree with your expectations? Explain.

4. Do you think that modeling planets as black bodies is a useful technique? Explain what is good and bad about using such a model.

5. What suggestions can you make as to how to improve this model?

6. Weather predictions are done with mathematical models, and some predictors do better jobs than others. Give some possible reasons why the Acme Weather Company (*not the company's real name*) has the worst record for predicting the weather.

Activity C

The Role of Actual Data in Mathematical Models

Overview

Models do not exist in a vacuum. They are based upon physical situations and require data as input in order to achieve desired output. The Excel spreadsheet model you built in the previous activity was based upon equations describing relationships observed in the real world. You determined these relationships by first making observations using physical models. You then provided data (luminosity of the sun and the distance of the planet from the sun) for the model to determine the surface temperature of a planet. The use of computer models allows us to extend our investigation beyond phenomena that can be modeled physically in the laboratory. Imagine trying to construct a physical model of the sun in order to better understand the mechanisms by which stars convert mass into energy. A mathematical representation is much easier to construct and infinitely safer to work with than a miniature sun.

How do we improve the mathematical descriptions used in our model? How do we obtain more data for fine tuning our model? This activity deals with these questions as we try to make our spreadsheet model better represent the characteristics of planets in our solar system.

You will use a more sophisticated spreadsheet model of a planet called *Global Equilibrium Energy Balance Interactive Tinker Toy* or GEEBITT. This model contains some of the modifications that you suggested would improve the spreadsheet models of your own design. GEEBITT takes into account the exact relationships between distance to the body at that point, as well as the reflectivity or albedo of a planet. The albedo of a planet can be input to GEEBITT along with the distance of the planet from the sun and the luminosity of the sun to determine the gray body surface temperature of the planet.

As a group, you will use NASA data from satellites to determine the average albedo of the earth. You will then use this albedo as an input to the GEEBITT Excel spreadsheet and determine a model value for the average surface temperature of the earth. Compare this theoretical value with the true value, and then perform similar comparisons for Mars and Venus, examples of cold and hot planets. Complete this activity by evaluating GEEBITT and determining further improvements that could be made to this mathematical model, if any.

Learning Objectives

- ✓ Read and analyze an albedo map of the earth.
- ✓ Determine the average albedo of the earth.
- ✓ Become familiar with and utilize the GEEBITT spreadsheet model.
- ✓ Compare theoretical and observed gray body temperatures for a planet.
- ✓ Suggest further improvements to the spreadsheet model.

Relevance

The quality of a model's output depends upon the accuracy of the mathematical descriptions used in the model, as well as the variety and accuracy of input data. If scientists are to make decisions and plan courses of action based upon the results of a model, they need to ensure that the model does the best job possible in simulating real world conditions. Real world data are a necessary component of this process and can be obtained from direct observations at the planet's surface or from observations via satellite. The data must be interpreted and converted into a usable form in order to be utilized by a model. This activity presents an example of such a process.

Investigation Activity C

The Role of Actual Data in Mathematical Models

Materials

Computers loaded with Microsoft Excel software. A maximum of 2-3 students per computer is suggested.
To use this spreadsheet you will need:
A set of Reference images of the earth's albedo maps
A copy of the spreadsheet Albedo Calculator
A copy of spreadsheet mini-GEEBITT Version A
(Both these spreadsheets can be downloaded from: http://icp.giss.nasa.gov/education/)

Methods

Preliminary Discussion and Preparation

1. Class discussion of the necessity of real data to serve as input into a model.
2. Review the concept of albedo with your classmates and instructor.
3. Interpret an albedo map presented by your instructor.

Interpreting Data and Using the Albedo Calculator

1. Determine the types and amounts of surfaces in the section of the earth assigned to your group.
2. Determine the approximate albedos of these sections.
3. Use the Albedo Calculator to determine the average albedo of this section of the earth.
4. Complete Experiment 1 – Part A: Determine the Average Albedo of the Earth.
5. Examine an albedo map of the same section of the earth that includes the influence of the atmosphere.
6. Use the Albedo Calculator to re-determine the average albedo of this section of the earth by including the atmospheric influence.
7. Complete Experiment 1 – Part B: Determine the Average Albedo of the Earth.

Experiment with the Model

1. Learn how to use the spreadsheet model of the solar system, GEEBITT. Read the GEEBITT User's Guide (see Appendix A).
2. Use GEEBITT and your average albedo to determine the surface temperature of the earth. Complete Experiment 2: Predicting Planetary Surface Temperatures.

Data Analysis and Consensus

1. Compare the output of your model with true values for Earth, Venus and Mars.
2. Evaluate the accuracy of GEEBITT.
3. Suggest ways of improving GEEBITT.
4. To assess what you learned in the modeling experiments, complete the Investigation Questions.

Investigation Notebook

1. Data Sheet: Experiment 1 – Part A: Determine the Average Albedo of the Earth
2. Data Sheet: Experiment 1 – Part B: Determine the Average Albedo of the Earth
3. Data Sheet: Experiment 2: Predicting Planetary Surface Temperatures
4. Questions: The Role of Actual Data in Mathematical Models
5. Essay: Considering a Real World Problem

Experiment 1 - Part A: Determine the Average Albedo of the Earth

Your instructor will divide the class into groups of six students. Each group will use the following reference materials provided for Activity C to conduct this study:

1. A satellite image of the earth. (*Figure 3.5, on page 97 with a key on page 98.*)
2. A world map of the albedo of the earth (created from NASA and NOAA satellite data), divided into six equal sections. (*On page 99.*)
3. A key showing the numerical values of the albedos. (*At the bottom of the albedo map.*)

Each group will be assigned one of the six regions of the earth. As a group, you are to determine the average albedo of your section.

Assigned section:

Tasks

1. Estimate the albedo of each type of surface identified in table 3.6 below.
2. Determine the number of grid boxes of each type of surface in your region and record these values in table 3.6.
3. Use these values in the Albedo Calculator spreadsheet to calculate the average albedo of your section.
4. Work with all the groups in your class to calculate an average albedo for Earth's surface.

Region	Total	Water	Land: Vegetation	Land: Desert	Land: Snow & Ice
Number of Area Units					
Percent (%) Area					
Percent (%) Surface Reflectivity					

Table 3.6. Albedo for your group's region of the earth's surface.

5. Enter the values of the estimated albedo of each type of surface and the percentage (in decimal format, 10% = 0.10) of that type of surface in your section of the earth into the Albedo Calculator spreadsheet. The spreadsheet will calculate the average albedo of your section. Record that result here:

Average albedo of your region:

6. Enter your average albedo as well as the average albedos of the other groups into table 3.7 below.
7. Determine the average albedo of earth from these values.

Region	Average Albedo
1	
2	
3	
4	
5	
6	
Average	

Table 3.7. Average albedo of the earth.

Analysis of Results

1. Do you think the result obtained for the average albedo of the earth is a good approximation of this value? Why or why not?

2. What might be done to make this approximation accurate?

Experiment 1 - Part B: Determine the Average Albedo of the Earth

Your instructor will now provide you with a map of the average albedo of the earth including the effects of its atmosphere. This map was created using NASA and NOAA satellite observations. (*Figure 3.8, Located on page 100.*)

Tasks

1. Determine the average albedo of your global region, but this time calculate the value when clouds and other atmospheric effects are included in the model.
2. Enter these values into table 3.8 below.

Region	Total	Water	Land: Vegetation	Land: Desert	Land: Snow & Ice
Number of Area Units					
Percent (%) Area					
Percent (%) Surface Reflectivity					

Table 3.8. Albedo including clouds and other atmospheric effects for your group's region of Earth's surface. Note: Percent surface reflectivity is adjusted for atmospheric effects.

3. Enter the results of the other groups for their sections into table 3.9 below.
4. Use these results to determine the average albedo of the entire earth if atmospheric effects are included.
5. Enter the results from table 3.9 into the Albedo Calculator spreadsheet to produce a value for the average albedo of the earth including the effects of clouds and other atmospheric features. Record that value below:

Average albedo of the earth including atmospheric effects:

Region	Average Albedo
1	
2	
3	
4	
5	
6	
Average	

Table 3.9. Average albedo of the earth including atmospheric effects.

Data Sheet Activity C

Experiment 2: Predicting Planetary Surface Temperatures

Your instructor will tell you when to use the GEEBITT spreadsheet. Once you have opened the spreadsheet, do the following. If necessary, refer to the GEEBITT user's guide in Appendix A.

1. Enter the values for luminosity of the sun and the average distance between the earth and the sun that you used when making your own model.
2. Use the spreadsheet to determine the surface temperature of the earth with these values.
3. Enter this temperature in the appropriate space in table 3.10 below.
4. Enter the value for the average albedo of the earth (including cloud effects) that you determined in the previous experiment. Be sure to use the decimal value for the albedo.
5. Use the spreadsheet to determine the theoretical surface temperature of the earth. Write that value in data table 3.10 below as well.
6. Repeat this activity using the average albedos of Venus (0.76), Mars (0.25) and Mercury (0.11) to determine the theoretical surface temperatures of these planets.
7. Enter the values for theoretical surface temperatures of these planets with and without albedos in table 3.10.

Planet	Average Surface Temperature Without Albedo	Average Surface Temperature With Albedo	Actual Average Surface Temperature
Earth			
Venus			
Mars			
Mercury			

Table 3.10. Theoretical average planetary surface temperatures.

Regional Effects of Human Development – Deforestation

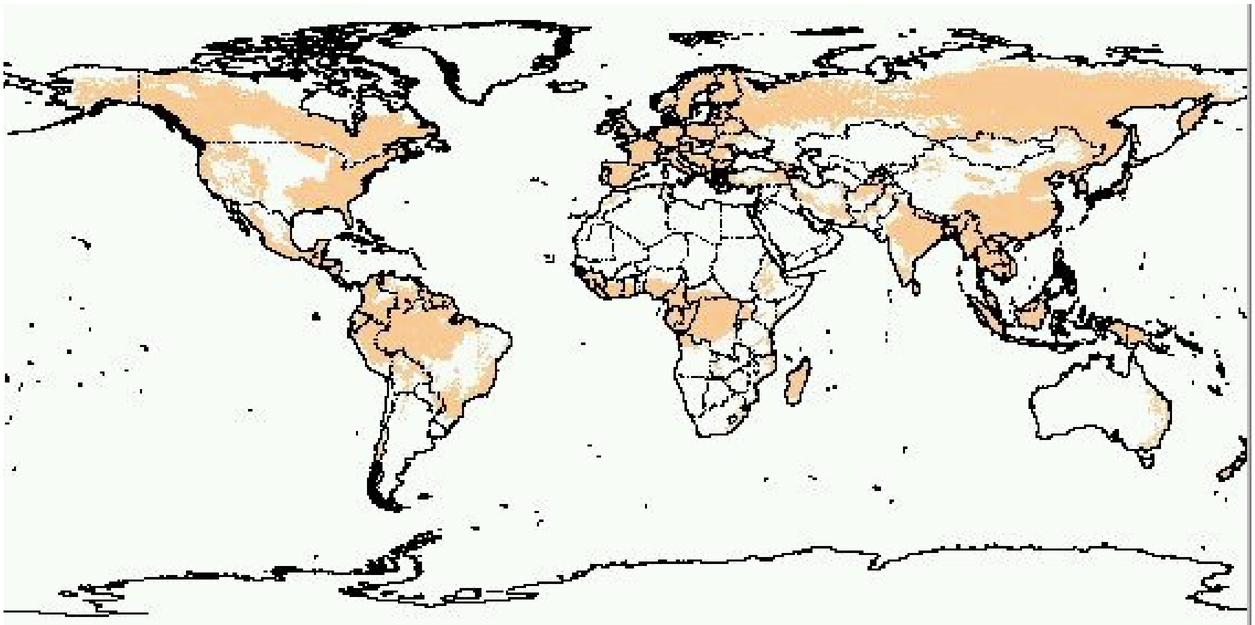


Figure 3.2. Forests on the earth 8000 years ago are indicated by the shaded areas.

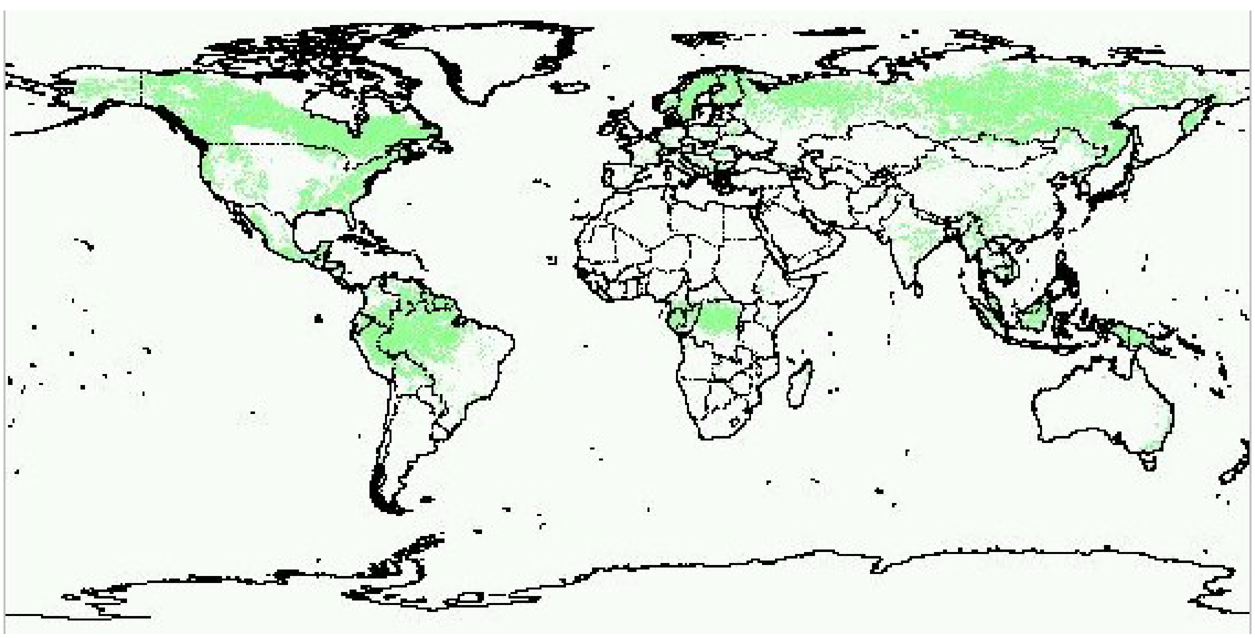


Figure 3.3. Current forests on the earth are indicated by the shaded areas.

Regional Effects of Human Development – Urban Heat Islands

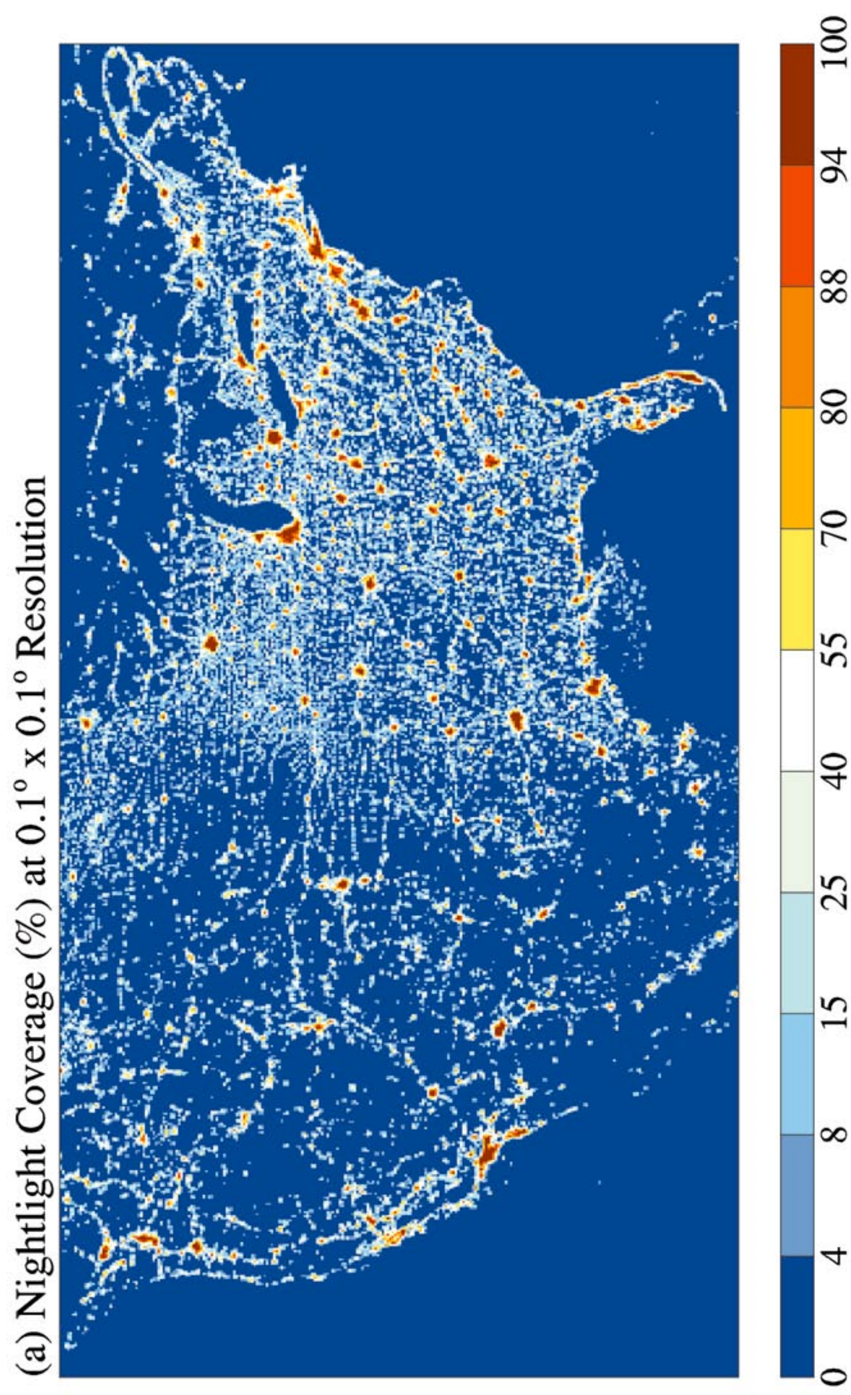


Figure 3.4. Satellite view of night time city lights in the United States. The brighter areas can be related to population density and land surface development.

Map of Earth's Surface Coverage

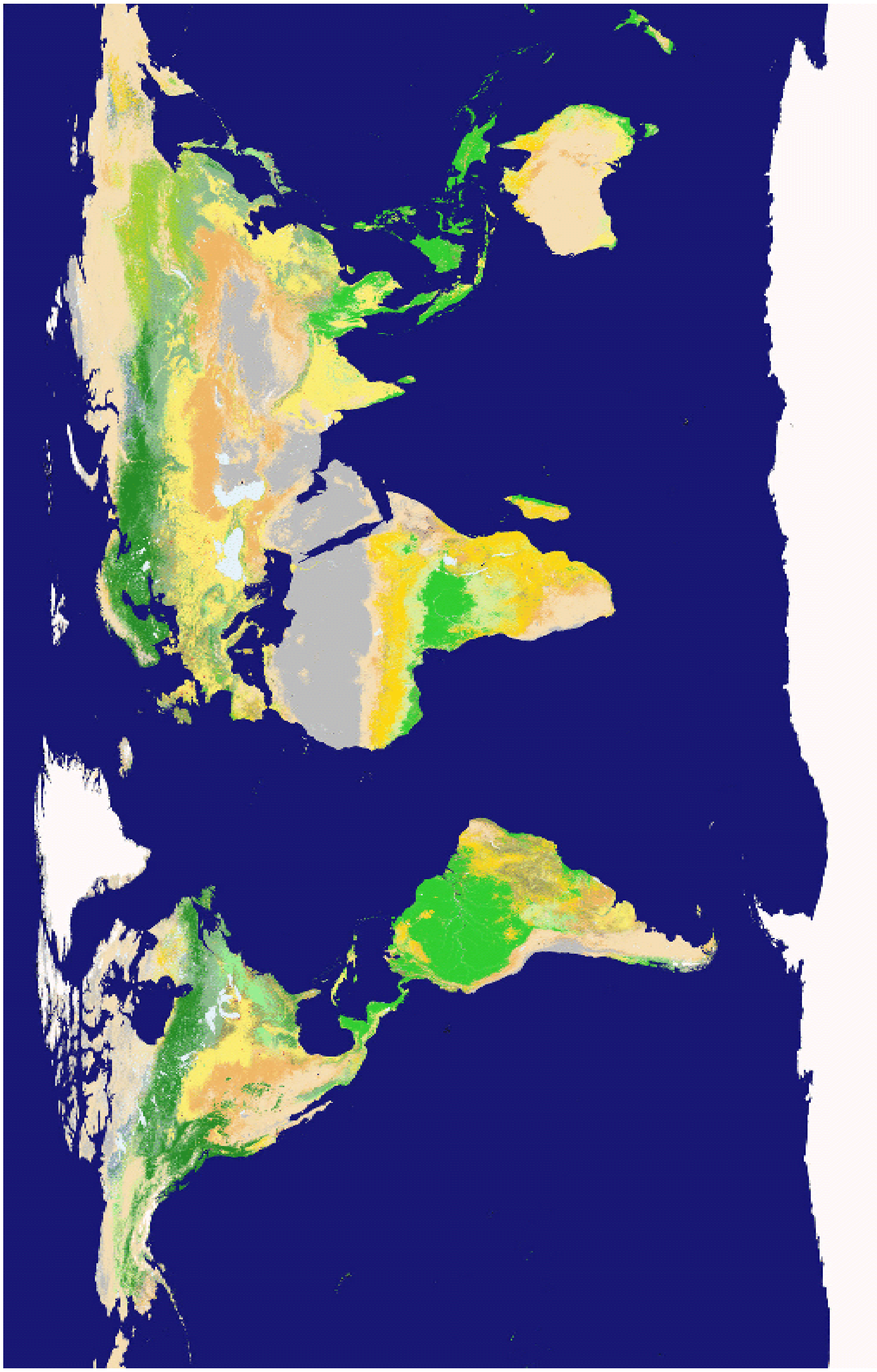


Figure 3.5. Map of Earth's Surface Coverage - October 2001. Data from the Moderate Resolution Imaging Spectroradiometer (MODIS) instrument on NASA's Terra satellite. The land cover maps were developed at Boston University in Boston, Mass.

Key for Earth's Surface Coverage Map

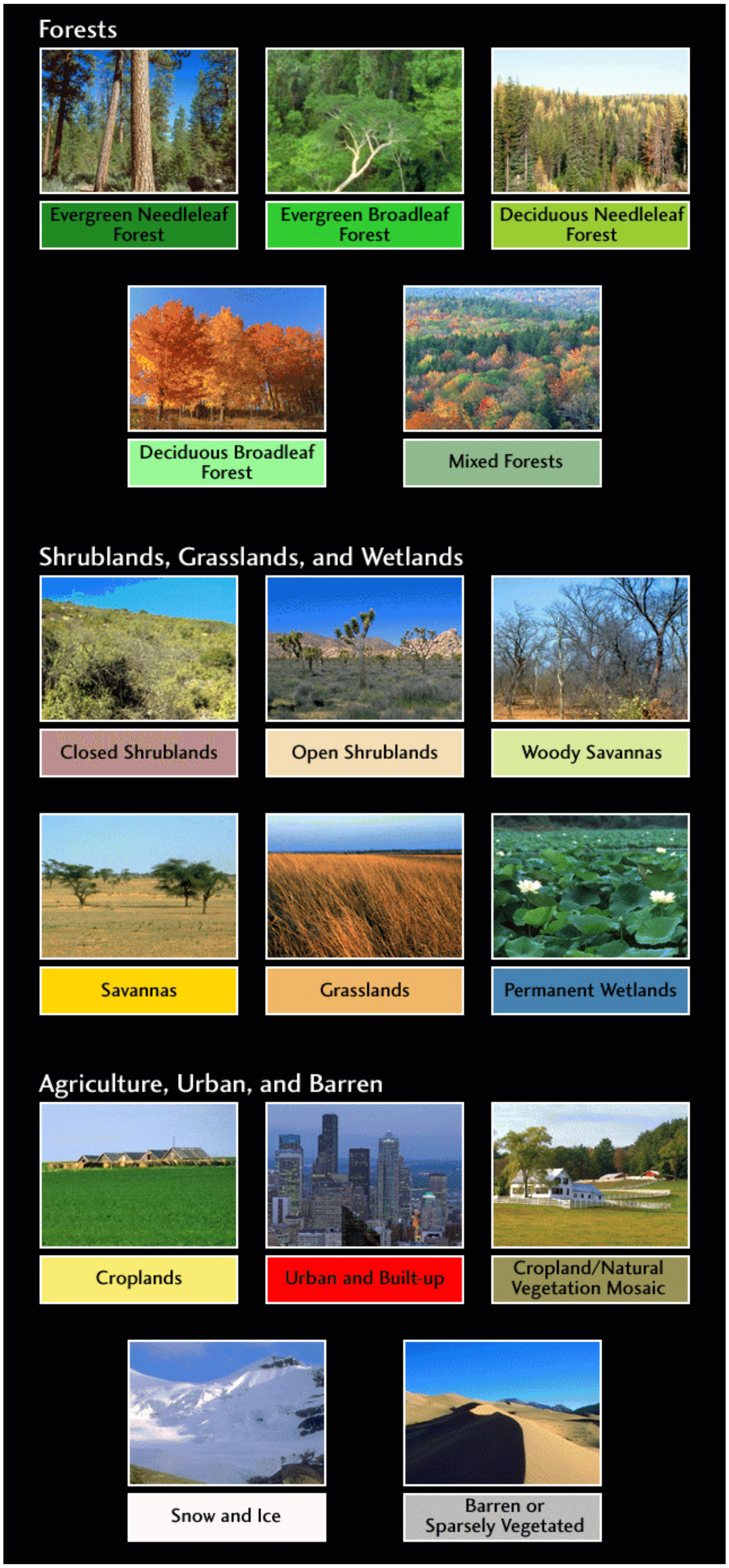


Figure 3.6. Key for Map of Earth's Surface Usage - October 2001. The Key and the land cover maps were developed at Boston University in Boston, Mass. (For more information and maps, go to http://earthobservatory.nasa.gov/Newsroom/LCC/

Map of Earth's Albedo

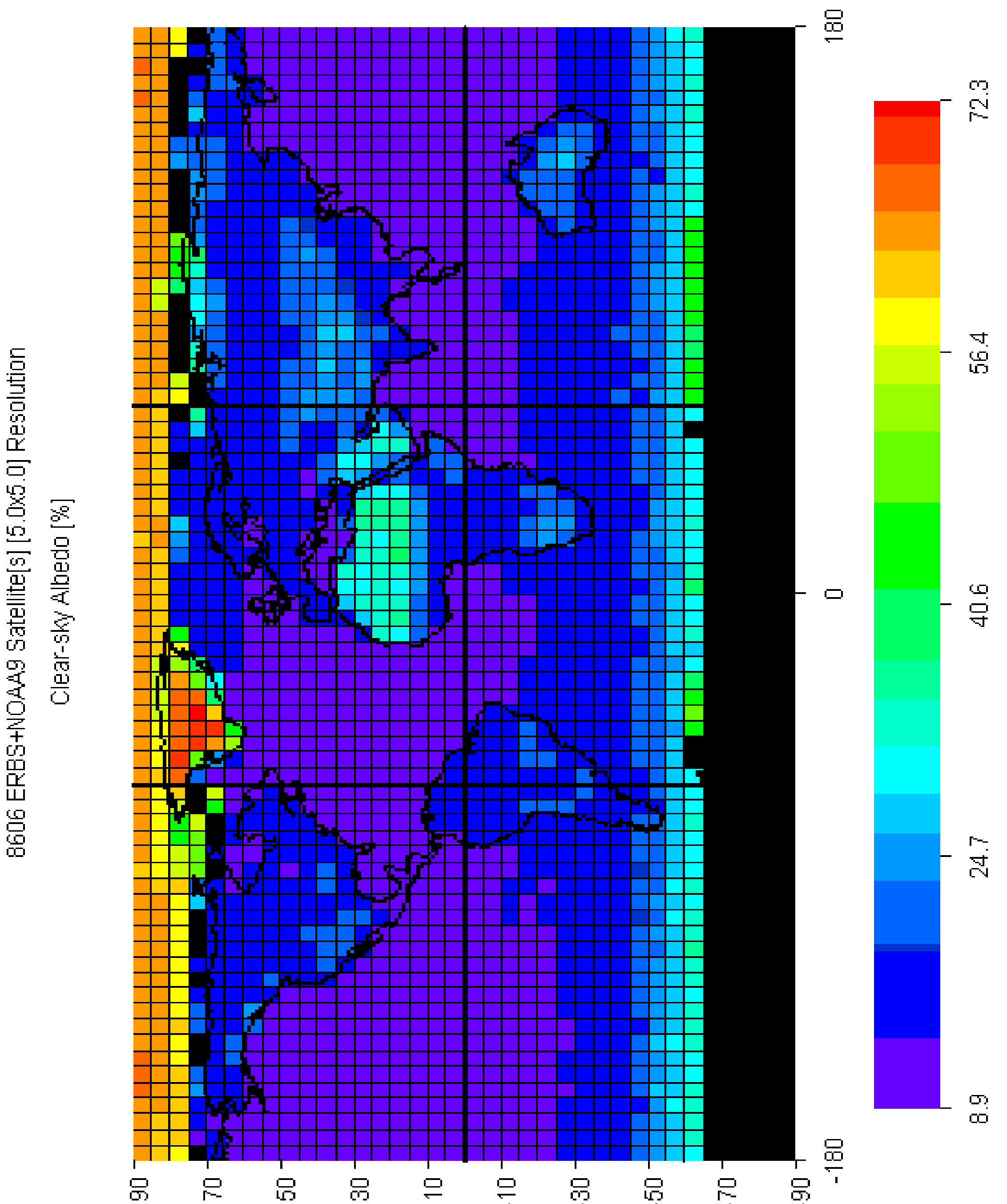


Figure 3.7. Map of Earth's Albedo – June 1986. Data source: NASA Earth Radiation Budget Experiment and NOAA-9 Satellites.

Map of Earth's Albedo with Sky Effects in June 1986

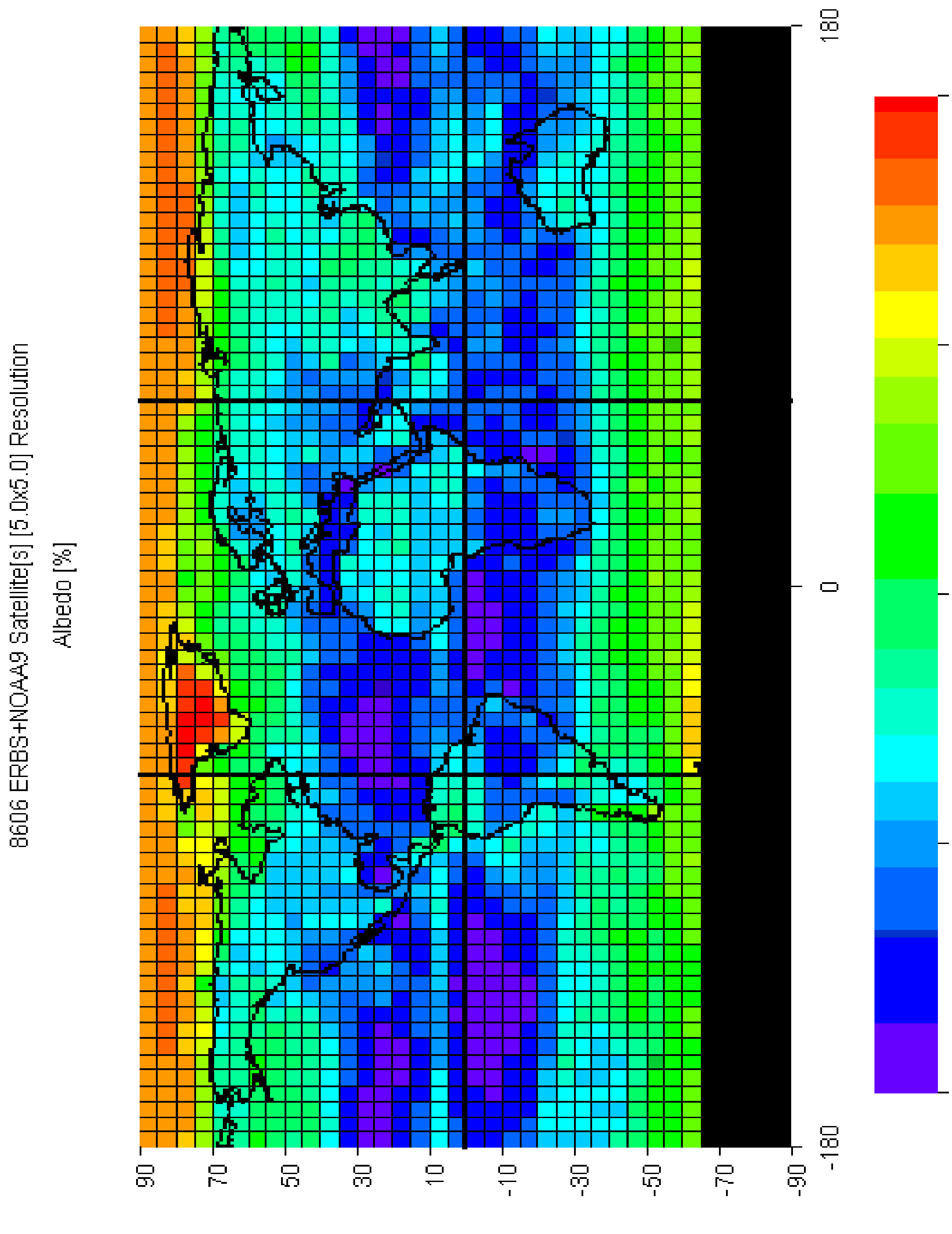


Figure 3.8. Map of Earth's Albedo (Non-Clear Sky) – June 1986. Data source: NASA Earth Radiation Budget Experiment and NOAA-9 Satellites.

Map of Earth's Albedo with Clear Sky in January 1986

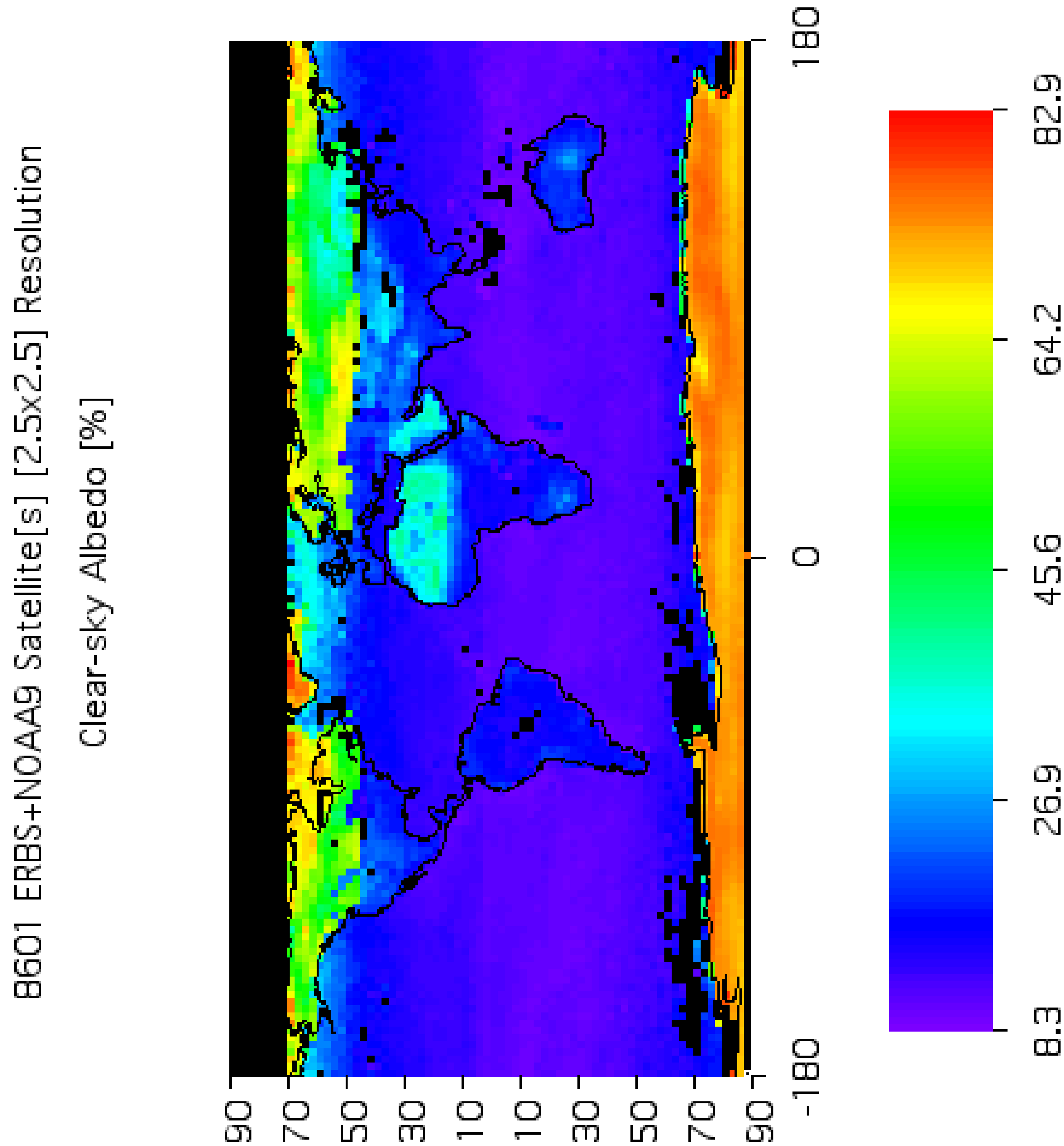


Figure 3.9. Map of Earth's Clear-sky Albedo – January 1986. Data source: NASA Earth Radiation Budget Experiment and NOAA-9 Satellites.

Map of Earth's Albedo with Clear Sky in July 1986

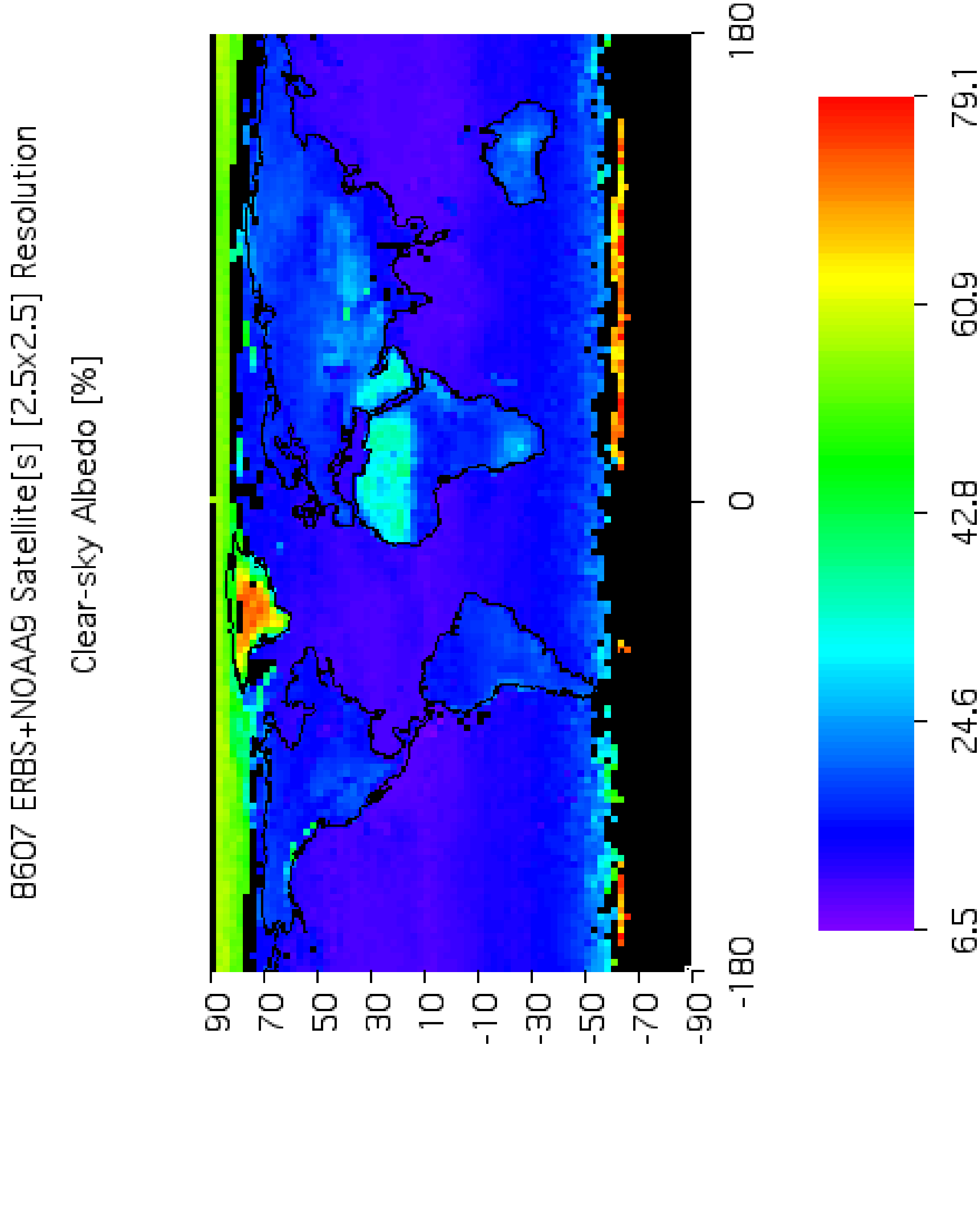


Figure 3.10. Map of Earth's Clear-sky Albedo – July 1986. Data source: NASA Earth Radiation Budget Experiment and NOAA-9 Satellites.

Questions Activity C

The Role of Actual Data in Mathematical Models

Answer the following questions on your own in complete sentences. You will need to use additional sheets of paper to answer all the questions completely.

1. How does the value for the average surface temperature of the earth without albedo compare with the value you obtained with your own spreadsheet model? Is this value what you would expect?

2. Compare the value of the surface temperature of the earth without albedo, with the average surface temperature with albedo. Is this what you would expect? Explain your reasoning.

3. Examine and compare the References: Map of Earth's Albedo with Clear-sky for January (*Figure 3.1.9, page 101*) and July (*Figure 3.1.10, page 102*). Answer the questions below.

   a. What similarities do you see between the two maps? Are these expected? Explain.

   b. What are the major differences between the two maps? Explain why you think these differences occur.

   c. In the past, the earth has been immersed in a series of Ice Ages. What changes would you expect to see in an albedo map of the earth during one of these ice ages?

4. What effect, if any, do you think the presence of humanity has upon the albedo of the earth? Explain your answer.

5. The actual average temperature at the earth's surface is 15ºC, that of Venus is 430ºC and that of Mars is – 45ºC. How do the theoretical values determined by GEEBITT compare with the true values? What does this indicate about the GEEBITT model?

6. Do you think it is possible to produce a model that is an exact simulation of the real world? Defend your answer.

Essay Topic 3

Deforestation and Urban Heat Islands

Write a 300 word scientific essay that addresses temperature changes as a result of variations in albedo on the earth's surface and atmosphere. (Refer to this topic's Real World Problem on page 60). How will removing forests and replacing them with agriculture and/or pastures affect albedo? Do you think large scale deforestation in the Amazon will significantly influence local and global temperatures? Would this cause warming or cooling? What are other important factors besides albedo that control temperature?

On a smaller scale, cities replace vegetation with buildings, producing Urban Heat Islands. Why does the city warm as reflectivity increases? Justify your responses with evidence from your Albedo Calculator and GEEBITT experiments. Include examples found on the Internet of areas where humans may be changing albedo and temperature. Use the Reference: Regional Effects of Human Development – Deforestation. (*Figures 3.2 & 3.3 on page 95*)